

Hong Kong Mathematics Olympiad (1990 – 91)

Heat Event (Group)

香港數學競賽 (1990 – 91)

初賽項目 (團體)

1. Find the unit digit of 1357^{7890} .
求 1357^{7890} 的個位數。

2. If $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \cdots + \frac{1}{2450} = \frac{x}{100}$, find x .
若 $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \cdots + \frac{1}{2450} = \frac{x}{100}$, 求 x 。

3. $\frac{a}{3}$, $\frac{b}{4}$ and $\frac{c}{6}$ are three proper fractions in their simplest form, where a , b and c are positive integers. If c is added to the numerator of each fraction, then the sum of the fractions formed will be equal to 6. Find the value of $a+b+c$.
 $\frac{a}{3}$ 、 $\frac{b}{4}$ 及 $\frac{c}{6}$ 是三個化至最簡的真分數，其中 a 、 b 及 c 是正整數。如果這三個分數的分子都加上 c ，則所得三個分數的和是 6。求 $a+b+c$ 的值。

4. Study the Pascal's triangle shown below:

Row 1					1				
Row 2					1		1		
Row 3				1		2		1	
Row 4			1		3		3		1
Row 5		1		4		6		4	1
Row 6	1		5		10		10		5
					⋮				

⋮

Find the sum of all the numbers from Row 1 to Row 15.

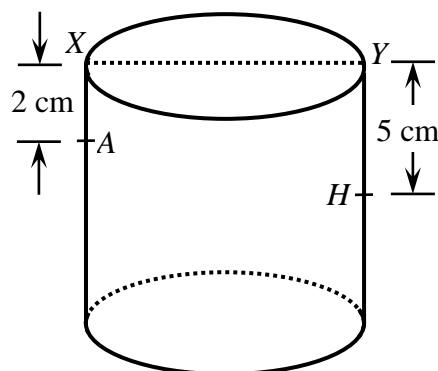
細讀下列之帕斯卡三角形

第 1 行					1					
第 2 行					1		1			
第 3 行				1		2		1		
第 4 行			1		3		3		1	
第 5 行		1		4		6		4	1	
第 6 行	1		5		10		10		5	1
					⋮					

⋮

求由第 1 行至第 15 行所有數的總和。

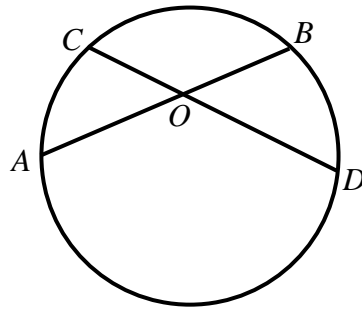
5. In the multiplication $\square\square\square \times \square\square = \square\square \times \square\square = 5568$, each of the above boxes represents an integer from 1 to 9. If the integers for the nine boxes above are all different, find the number represented by $\square\square\square$.
- 在下列乘法算式中
- $$\square\square\square \times \square\square = \square\square \times \square\square = 5568,$$
- 每一方格代表由 1 至 9 的一個整數。若以上九個方格所代表的九個整數都不相同，求 $\square\square\square$ 所代表的整數。
6. Find the remainder when $1997^{1990} - 1991$ is divided by 1996.
- 求 $1997^{1990} - 1991$ 被 1996 除所得的餘數。
7. Find the least positive integral value of n such that $\sqrt{n} - \sqrt{n-1} < \frac{1}{80}$.
- 求滿足不等式 $\sqrt{n} - \sqrt{n-1} < \frac{1}{80}$ 的 n 的最小正整數值。
8. One of the solutions of the equation $32x + 59y = 3259$ in positive integers is given by $(x, y) = (100, 1)$. It is known that there is exactly one more pair of positive integers a, b ($a \neq 100$ and $b \neq 1$) such that $32a + 59b = 3259$. Find a .
- 方程 $32x + 59y = 3259$ 的其中一組正整數解為 $(x, y) = (100, 1)$ 。現知僅有另一組正整數 a, b ($a \neq 100, b \neq 1$) 使得 $32a + 59b = 3259$ ，求 a 。
9. In Figure 1, XY is a diameter of a cylindrical glass, 48 cm in base circumference. On the outside is an ant at A , 2 cm below X and on the inside is a small drop of honey at H , 5 cm below Y . If the length of the shortest path for the ant to reach the drop of honey is x cm, find x . (Neglect the thickness of the glass.)
- 在圖 1 中， XY 是圓柱形玻璃杯的直徑，杯底的圓周是 48 cm。杯外 A 點處 (在 X 之下 2 cm) 有一蟻，杯內 H 點處 (在 Y 之下 5 cm) 有一小滴蜜糖。若蟻行至蜜糖的最短路線長 x cm，求 x 。(杯的厚度可略去不計。)



(Figure 1)(圖 1)

10. In Figure 2, two chords AOB , COD cut at O . If the tangents at A and C meet at X , the tangents at B and D meet at Y and $\angle AXC = 130^\circ$, $\angle AOD = 120^\circ$, $\angle BYD = k^\circ$, find k .

在圖 2 中，弦 AOB 、 COD 相交於 O 。若過 A 的切線與過 C 的切線相交於 X ，過 B 的切線與過 D 的切線相交於 Y ，且 $\angle AXC = 130^\circ$ 、 $\angle AOD = 120^\circ$ 、 $\angle BYD = k^\circ$ ，求 k 。



(Figure 2)(圖 2)